Joint workshop on analysis of PDEs

Shanghai Jiao Tong University, China & Sharif University of Technology, Iran

2022-07-04

1 Tencent Meeting (Voov Meeting) Information

Meeting Topic: Joint workshop on analysis of PDEs
Meeting Time : 2022/07/04

14:30-18:00 China Standard Time - Beijing

11:00-14:30 Iran Standard Time - Tehran

08:30-12:00 Sweden Standard Time - Stockholm

Meeting ID: 737 695 182
Meeting Password: 0704
Meeting Link: https://meeting.tencent.com/dm/4qHjNZvPjv4c

2 Meeting Schedule

Session I: 14:30–16:00

1. **14:30–15:00 Henrik Shahgholian, KTH**
   Free boundary: a general perspective.
   Chair: Yachun Li, SJTU

2. **15:00–15:30 Deng Zhang, SJTU**
   Sharp non-uniqueness for the 3D hyperdissipative Navier-Stokes equations:above the Lions exponent.
   Chair: Morteza Fotouhi, SUT

3. **15:30–16:00 Yue Cao, SJTU**
   Local strong solutions to the full compressible Navier-Stokes system with temperature-dependent viscosity and heat conductivity.
   Chair: Chunjing Xie, SJTU

Tea Break: 16:00–16:10

Session II: 16:10–17:40

1. **16:10–16:40 Chunjing Xie, SJTU**
   Analysis of steady solutions for the incompressible Euler system in an infinitely long nozzle.
   Chair: Henrik Shahgholian, KTH

2. **16:40–17:10 Erfan Salavati, Amirkabir University of Technology**
   Optimal Control of Stochastic Differential Equations Driven by G-Brownian Motion.
   Chair: Deng Zhang, SJTU

3. **17:10-17:40 Zirong Zeng, SJTU**
   Non-uniqueness of weak solutions to 3D magnetohydrodynamic equations.
   Chair: Erfan Salavati, Amirkabir University of Technology

Session III: 16:40–18:00 Free Discussions

3 Abstracts

1. Yue Cao, SJTU
   Title: Local strong solutions to the full compressible Navier-Stokes system with temperature-dependent viscosity and heat conductivity
   Abstract: We study the full compressible Navier-Stokes system in a bounded domain $\Omega \subset \mathbb{R}^3$, where the viscosity and heat conductivity depend on temperature in a power law (θ^b for some constant $b > 0$) of Chapman-Enskog. We obtain the local existence of strong solution to the initial-boundary value problem, which is not trivial especially for the non-isentropic system with vacuum and temperature-dependent viscosity. There are degeneracy caused by vacuum and extremely strong nonlinearity caused by variable coefficients, which bring great difficulty to the a priori estimates, especially for the second-order estimates. First, in order to obtain closed first-order estimates, we introduce a new variable to reformulate the system into a better form and require the measure of initial vacuum domain is sufficiently small. Second, with the help of cut-off and straightening out technique, and the thermo-insulated boundary condition, we establish time involved estimate for the second-order derivative of temperature, which plays a key role in closing the a priori estimates. Moreover, our local existence result holds for the cases that the viscosity and heat conductivity depend on θ with possibly different power laws (i.e., $\mu, \lambda \sim \theta^{b_1}$, $\kappa \sim \theta^{b_2}$ with constants $b_1, b_2 \in [0, +\infty)$). This is a joint work with Yachun Li.

2. Erfan Salavati, Amirkabir University of Technology
   Title: Optimal Control of Stochastic Differential Equations Driven by G-Brownian Motion
   Abstract: In physical and social phenomena, the level of uncertainty that one has to assume in a model, depends both on the preferences and the information available to the decision maker. There are two widely used levels of uncertainty in the literature. The first one is called risk and represents the inherent randomness present in the phenomena. Modeling of risk is done using probability distributions. Usually a parametric family of distributions is assumed and then the parameters are estimated using classical statistical procedures. The estimated parameters are then used as inputs to the optimization problem which arise in decision making. The second level of uncertainty is called ambiguity and represents the uncertainty that arises

from the lack of information about the parameters of the model. Modeling of ambiguity is done using worst-case analysis over all possible values of the parameters of the model. In the context of stochastic processes, the idea of ambiguity has been applied to the Brownian motion and gave rise to the concept if G-Brownian motion. This process can be thought if as a Brownian motion whose diffusion coefficient is ambiguous. This new paradigm is not achievable through the classical probability theory, and for this reason a new theory of probability has been developed known as non-linear expectation (in contrast with classical expectation which is linear). In this new theory, almost every concept of classical stochastic analysis finds its G-counterpart. Examples are G-martingales, G-Ito's formula, G-SDE and ... In this project, we consider a controlled stochastic differential equations driven by G-Brownian motion as $dX_t = a(t, X_t, u_t)dt + b(t, X_t, u_t)dB_t + c(t, X_t, u_t)d < B >_t$, where B_t is G-Brownian motion and $< B >_t$ is its quadratic variation. We associate the following payoff to this SDE, $J(u(\cdot)) = \int_0^T f(t, X_t, u_t)dt + g(X_T)$, and seek the control policy u which maximizes J. We wish to study the properties of the value function of the above optimal control problem. Specifically, we will study the solution concepts of the Hamilton-Jacobi-Bellman-Isaacs (HJBI) equation which the value function satisfies.

3. Henrik Shahgholian, KTH

Title: Free boundary: a general perspective

Abstract: This talk is an overview on free boundary problems and techniques.

4. Chunjing Xie, SJTU

Title: Analysis of steady solutions for the incompressible Euler system in an infinitely long nozzle

Abstract: Stagnation point in flows is an interesting phenomenon in fluid mechanics. It induces many challenging problems in analysis. We first derive a Liouville type theorem for Poiseuille flows in the class of incompressible steady inviscid flows in an infinitely long strip, where the flows can have stagnation points. With the aid of this Liouville type theorem, we show the uniqueness of solutions with positive horizontal velocity for steady Euler system in a general nozzle when the flows tend to the horizontal velocity of Poiseuille flows at the upstream. Finally, this kind of flows are proved to exist in a large class of nozzles. This is a joint work with Congming Li and Yingshu Lv.

5. Zirong Zeng, SJTU

Title: Non-uniqueness of weak solutions to 3D magnetohydrodynamic equations

Abstract: In this talk, we consider the non-uniqueness of weak solutions to 3D MHD equations. The constructed weak solutions do not conserve the magnetic helicity and can be close to any given smooth, divergence-free and mean-free velocity and magnetic fields. Furthermore, for weak solutions in some spaces to the ideal MHD, we prove that they are the strong vanishing viscosity and resistivity limit of the weak solutions to MHD equations. This shows that, in contrast to the weak ideal limits, Taylor's conjecture does not hold along the vanishing viscosity and resistivity limits. More interestingly, we are able to prove the aforementioned results for the hyper-viscous and hyper-resistive MHD equations up to the sharp exponent 5/4, which coincides exactly with the Lions exponent for 3D hyper-viscous NSE. This work is in joint with Yachun Li and Deng Zhang.

6. Deng Zhang, SJTU

Title: Sharp non-uniqueness for the 3D hyperdissipative Navier-Stokes equations: above the

Lions exponent

Abstract: It is well-known that, due to Lions, for any L2 divergence-free initial data, there exist unique smooth Leray-Hopf solutions when the viscosity exponent is larger than 5/4. We prove that even in this high dissipative regime, the uniqueness would fail in the supercritical mixed Lebesgue spaces, in view of the generalized Ladyzenskaja-Prodi-Serrin condition. The non-uniqueness is proved in the strong sense and, in particular, yields the sharpness at two endpoints. Furthermore, the constructed non-unique weak solutions can be close to the Leray-Hopf solutions in the superciritical spaces and admit the partial regularity outside a fractal set of singular times with small Hausdorff dimension. This work is in joint with Yachun Li, Peng Qu and Zirong Zeng.